Lösungen zu Extremwertaufgaben II

1. (a) $V(x) = x(1\text{m} - 2x)^2 = 1\text{m}^2 x - 4\text{m}x^2 + 4x^3$

   (b) $V'(x) = 1\text{m}^2 - 8\text{m}x + 12x^2 = 0 \Leftrightarrow x_1 = \frac{1}{2}\text{m}$ oder $x_2 = \frac{1}{6}\text{m}$
   $V(0) = V(\frac{1}{2}\text{m}) = 0$.
   $V''(x) = -8\text{m} + 24x \Rightarrow V''(\frac{1}{6}\text{m}) = -4\text{m} < 0 \Rightarrow$
   Maximum bei $x_2 = \frac{1}{6}\text{m}$.

   (c) $V(\frac{1}{6}\text{m}) = \frac{2}{27}\text{m}^3$

2. (a) $V(x) = x(a - 2x)^2 = a^2 x - 4ax^2 + 4x^3$

   (b) $V'(x) = a^2 - 8ax + 12x^2 = 0 \Leftrightarrow x_1 = \frac{1}{2}a$ oder $x_2 = \frac{1}{6}a$
   $V(0) = V(\frac{1}{2}a) = 0$.
   $V''(x) = -8a + 24x \Rightarrow V''(\frac{1}{6}a) = -4a < 0 \Rightarrow$
   Maximum bei $x_2 = \frac{1}{6}a$.

   (c) $V(\frac{1}{6}a) = \frac{2}{27}a^3$

3. (a) $V(x) = x(a - 2x)(b - 2x) = abx - 2(a + b)x^2 + 4x^3$

   (b) $V'(x) = 12x^2 - 4(a + b)x + ab = 0 \Leftrightarrow$
   $x_1 = \frac{1}{6}(a + b + \sqrt{a^2 + b^2 - ab})$ oder $x_2 = \frac{1}{6}(a + b - \sqrt{a^2 + b^2 - ab})$
   $V''(x) = -4(a + b) + 24x \Rightarrow$
   $V''(x_1) = 4\sqrt{a^2 + b^2 - ab} > 0 \Rightarrow$ Minimum bei x_1
   $V''(x_2) = -4\sqrt{a^2 + b^2 - ab} < 0 \Rightarrow$ Maximum bei x_2.

4. (a) $f'(x) = 4x^3 - 4x = 0 \Rightarrow x_1 = 0, \ x_{2/3} = \pm 1$

   (b)

   (c) $A'(x) = (x \cdot f(x))' = 5x^4 - 6x^2 + 1 = 0 \Rightarrow$
   $x_{1/2} = \frac{1}{10}(6 \pm \sqrt{36 - 20}) = \frac{1}{10}(6 \pm 4) \Rightarrow x_1 = 1, \ x_2 = \frac{1}{5}\sqrt{5}$.
   $A''(x) = 20x^3 - 12x$,
   $A''(1) > 0$, also Minimum.
   $A''(\frac{1}{5}) = -\frac{8}{5}\sqrt{5} < 0$, also Maximum. Also: $P\left(\frac{1}{5}\sqrt{5} \,\middle|\, 0{,}64\right)$